## **Mathematical Modeling and Analysis**



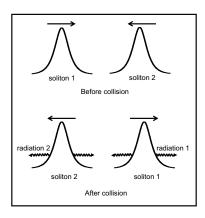
## Inelastic collisions between solitons in optical fibers

Avner Peleg avner@cnls.lanl.gov Misha Chertkov chertkov@lanl.gov Ildar Gabitov gabitov@math.arizona.edu

In fiber optics communications systems optical pulses are used as bits of information. A sequence of pulses launched into a fiber forms a pattern that codes the transmitted message. Each pulse in the sequence is positioned in the center of a slot allocated for the information bit. State "1" is assigned to a slot if the pulse is present there, and state "0" if the slot does not contain a pulse. Ideally, the pulse sequences propagate without any changes. However, certain physical processes break this ideal picture and lead to deterioration of the bit pattern and to loss of information.

Modern high-speed fiber optics communications systems extensively use multiple frequency channels for transmission of information. In each frequency channel pulses move with the same group velocity, but the group velocity is different for different channels. Therefore, collisions between pulses from different channels (interchannel collisions) are very frequent. These interchannel collisions can lead to the deterioration of the optical pulses in a variety of ways, and as a result, pose a major limitation on the performance of fiber optics transmission systems. With the increasing demand for faster transmission of information one can expect a corresponding increase in the number of channels used. As a result, the importance of interchannel collisions between optical pulses is expected to increase, and an accurate description of the effects of these collisions is needed.

It is known that propagation of electromagnetic pulses in optical fibers is described by the nonlinear Schrödinger equation (NLSE). Solitons are solutions of the NLSE which are particularly important due to their special properties: they are localized (i.e., pulse-like), stable, and stationary (i.e., they propagate without any change in their shape). Physically, this means that solitons are



Schematic description of the collision between two solitons from different frequency channels. The straight arrows denote the group velocity of the solitons, and the curly arrows denote radiation emitted by each of the two solitons in its own frequency channel.

localized pulses that propagate along the optical fiber without changing their shape and without emitting radiation, and that they are stable against small changes in their shape. Due to these properties, solitons are ideal candidates for transmission of information in fiber optics communication systems.

In this paper we study interchannel collisions between optical pulses using optical solitons as an example. In an ideal fiber interchannel collisions between solitons are elastic, that is, the amplitude, frequency, and shape of the solitons are not changed by the collision. In addition, no radiation is emitted due to the collisions. In real optical fibers, however, this ideal elastic nature of soliton collisions breaks down due to the presence of high order corrections (perturbations) to the ideal NLSE. In this case collisions between solitons from different frequency channels might lead to emission of radiation, change in the soliton amplitude and frequency, corruption of the soliton shape, and other undesirable effects. (See Figure 1 for a schematic description of the collision process.)

In this paper we focus our attention on the ef-

## Inelastic collisions between solitons in optical fibers

fect of third order dispersion, which is the slope of the dispersion as a function of frequency. The effect of third order dispersion on inter-channel soliton collisions is expected to be dominant compared to other high order effects near the wavelength in which the dispersion coefficient becomes zero. We calculate the dynamics and the total intensity of the continuous radiation emitted as a result of the collision. We also calculate the change, induced by the collision, in the soliton parameters. To achieve these goals we first find a stationary single-pulse solution taking into account third order dispersion. We then use two such stationary solutions as the initial condition for the collision problem. To find the effects of the collision we develop a perturbation theory with respect to two small parameters: the third order dispersion coefficient and the reciprocal of the frequency difference between the two channels.

We find that the amplitude of the emitted radiation is proportional to the third order dispersion coefficient divided by the square of the frequency difference between the channels. Moreover, the radiation emitted in the collision can

be described as resulting from a fast change in the dispersion coefficient. The amplitude and the frequency of the solitons do not acquire any change up to the third order of the perturbation theory. Using the results for a single collision we showed that soliton propagation in a given frequency channel influenced by many collisions with a random sequence of solitons from all other channels is equivalent to soliton propagation in fibers with weak disorder in the second order dispersion coefficient. As a result, one can expect the emergence of long-range interaction between solitons propagating in the same frequency channel, which is mediated by the emitted radiation. This long-range interaction leads to a severe random walk of the solitons, due to which the solitons walk out of their assigned slots and information is lost.

## Acknowledgements

Los Alamos Report LA-UR-02-5716. Funded by the Department of Energy under contracts W-7405-ENG-36 and the DOE Office of Science Advanced Computing Research (ASCR) program in Applied Mathematical Sciences.